158. Proposed by R. D. BOHANNON, Ph. D., Professor of Mathematics, Ohio State University, Columbus, O.

If $\dfrac{x}{a+\alpha}+\dfrac{y}{b+\alpha}+\dfrac{z}{c+\alpha}=\dfrac{x}{a+\beta}+\dfrac{y}{b+\beta}+\dfrac{z}{c+\beta}=\dfrac{x}{a+\gamma}+\dfrac{y}{b+\gamma}+\dfrac{z}{c+\gamma}=1$, show,

$$\frac{x}{(a+\beta)^2}+\frac{y}{(b+\beta)^2}+\frac{z}{(c+\beta)^2}=\frac{(\gamma-\beta)(\alpha-\beta)}{(a+\beta)(b+\beta)(c+\beta)}.$$

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Consider the equation,

$$\frac{x}{a+\varphi}+\frac{y}{b+\varphi}+\frac{z}{c+\varphi}=1-\frac{(\varphi-\alpha)(\varphi-\beta)(\varphi-\gamma)}{(a+\varphi)(b+\varphi)(c+\varphi)}.$$

Multiply through by $a+\varphi$ and then put $a+\varphi=0$.

$$\therefore x=\frac{(a+\alpha)(a+\beta)(a+\gamma)}{(a-b)(a-c)},\quad y=\frac{(b+\alpha)(b+\beta)(b+\gamma)}{(b-c)(b-a)},\quad z=\frac{(c+\alpha)(c+\beta)(c+\gamma)}{(c-a)(c-b)}$$

$$\therefore \frac{x}{(a+\beta)^2}+\frac{y}{(b+\beta)^2}+\frac{z}{(c+\beta)^2}=\frac{(a+\alpha)(a+\gamma)}{(a+\beta)(a-b)(a-c)}$$

$$+\frac{(b+\alpha)(b+\gamma)}{(b+\beta)(b-c)(b-a)}+\frac{(c+\alpha)(c+\gamma)}{(c+\beta)(c-a)(c-b)}=\frac{(\gamma-\beta)(\alpha-\beta)}{(a+\beta)(b+\beta)(c+\beta)}.$$

Also $\dfrac{x}{(a+\alpha)^2}+\dfrac{y}{(b+\alpha)^2}+\dfrac{z}{(c+\alpha)^2}=\dfrac{(\gamma-\alpha)(\beta-\alpha)}{(a+\alpha)(b+\alpha)(c+\alpha)}.$

$$\frac{x}{(a+\gamma)^2}+\frac{y}{(b+\gamma)^2}+\frac{z}{(c+\gamma)^2}=\frac{(\alpha-\gamma)(\beta-\gamma)}{(a+\gamma)(b+\gamma)(c+\gamma)}.$$

Also solved by *LON C. WALKER*.

159. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

If $x-1=3m$, $x^2-1=4n$, $x^3-1=5p$, where m, n, p are integers, find a general expression for x.

I. Solution by L. E. DICKSON, Ph. D., The University of Chicago, Chicago, Ill.

The required expression for x is of the form $3m+1$, where m is subject to the conditions that $(3m+1)^2-1$ shall be divisible by 4, and that $(3m+1)^3-1$ shall be divisible by 5. The first condition is satisfied if, and only if, m is even. The second condition requires that either m or else $3m^2+3m+1$ shall be divisible by 5. To prove that the last alternative is impossible, we note that

$$2(3m^2+3m+1)\equiv(m+3)^2-2 \pmod 5),$$

and hence is not$\equiv 0$ (mod 5), 2 being a non-quadratic residue of 5. The necessary and sufficient conditions are, therefore, that m shall be divisible by 10, so that x shall have the form $x=30t+1$.

II. Solution by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

From the three equations we readily obtain

$$x-1=3m \qquad\qquad =3m,$$
$$x^2-1=3m(2+3m) \qquad =4n,$$
$$x^3-1=9m[1+3m(1+m)]=5p;$$
$$\text{whence } n=\tfrac{3}{4}m(2+3m) \text{ and } p=\tfrac{9}{5}m[1+3m(1+m)],$$

where m must be so taken as to make n and p integers. To make n integral m must be an *even* number; to make p integral m must be a multiple of 5; hence $m=10k$ where k may be any positive integer. Hence

$$m=10k,$$
$$n=15k(1+15k),$$
$$p=18k[1+30k(1+10k)].$$
$$\text{Also } x-1=30k,$$
$$x^2-1=60k(1+15k),$$
$$x^3-1=90k[1+30k(1+10k)];$$
$$\text{and } x=1,\ 31,\ 61,\ 91, \text{ etc.}$$

Solved similarly by *G. B. M. ZERR*, *LON C. WALKER*, and *HON. J. H. DRUMMOND*.

160. Proposed by J. SCHEFFER, A. M., Hagerstown. Mo.

Represent the square root of $10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}$ as the sum of three square roots.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; LON C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University; and the late HON. JOSIAH H. DRUMMOND.

Let $\sqrt{[10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}]}=\sqrt{x}+\sqrt{y}+\sqrt{z}$.

$\therefore\ 10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}=x+y+z+2\sqrt{xy}+2\sqrt{xz}+2\sqrt{yz}$.

$\therefore\ 2\sqrt{xy}=2\sqrt{6}$, $\therefore\ \sqrt{xy}=\sqrt{6}$. Similarly, $\sqrt{xz}=\sqrt{10}$, $\sqrt{yz}=\sqrt{15}$.

$\therefore\ \sqrt{(xyz)}=\sqrt{30}$. $\therefore\ \sqrt{x}=\sqrt{2}$, $\sqrt{y}=\sqrt{3}$, $\sqrt{z}=\sqrt{5}$.

$\therefore\ \sqrt{[10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}]}=\sqrt{2}+\sqrt{3}+\sqrt{5}=$ the sum of the three square roots.